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Exact results on massless 3-flavor QCD and $\mathbb{Z}_3$-QCD.

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Based on work w/ Y. Kikuchi, T. Misumi, N. Sakai.

1. Motivation

UV dynamics : QCD Lagrangian, well controled.

⇓

IR dynamics : chiral symmetry breaking, confinement. ...

very difficult.

At finite μ, numerical study is not available.

⇒ Find RG-invariant for QCD and discuss the possible behavior.

't Hooft anomaly

d-dim. QFT with a global symmetry G.

Consider the G-background gauge field A, and the partition func.

$$Z[A].$$

Under the G-gauge trans. $A \to A + \delta_\theta A$, the phase of partition func. can be ambiguous

$$Z[A+\delta_\theta A] = Z[A] \cdot \exp(i \underbrace{\mathcal{A}[\theta, A]}_{\text{local functional of } \theta \& A}).$$

If $\mathcal{A}[\theta, A]$ cannot be eliminated by the local counter term $\delta_\theta C[A]$, then $\mathcal{A}[\theta, A]$ is called an 't Hooft anomaly.

Classic example

N_f-flavor massless QCD has $SU(N_f)_L \times SU(N_f)_R$

$$Z[L+\delta_{\theta_L} L,\ R+\delta_{\theta_R} R] = Z[L,R] \cdot \exp\left(\frac{i}{24\pi^2}\int \mathrm{tr}\left[\theta_R\, d\left(R\,dR + \tfrac{1}{2}R^3\right) - \theta_L\, d\left(L\,dL + \tfrac{1}{2}L^3\right)\right]\right).$$

i.e. $SU(N_f)_L \times SU(N_f)_R$ has an 't Hooft anomaly.

Thm 't Hooft anomaly is RG-inv.

(Proof) By def., there's no d-dim. local term $C[A]$ s.t.

$$\mathcal{A}[\mathcal{O}, A] = \delta_{\mathcal{O}} C[A].$$

However, the anomaly-inflow by $(d+1)$-dim. action $S_{d+1}[A]$ is possible:

$$\mathcal{A}[\mathcal{O}, A] = \delta_{\mathcal{O}} S_{d+1}[A].$$

(For chiral symmetry, this is the anomaly descent procedure).

When this is possible, we can define the problem in a Gauge inv. way:

$$\mathcal{Z}[A] \cdot \exp(-i S_{d+1}[A]).$$

Consider the RG flow. In the $(d+1)$-dim. bulk, the state is unique because this is the SPT phase. Thus, the RG only eliminates the massive boundary excitations.

$\Rightarrow$ The vacua & massless excitation must match the anomaly $\delta_{\mathcal{O}} S_{d+1}[A] = \mathcal{A}[\mathcal{O}, A]$.

$\therefore \mathcal{A}[\mathcal{O}, A]$ is RG-inv. //

Short summary

We'll find a new 't Hooft anomaly for massless QCD.

$$G = \frac{SU(N_f)_V}{\mathbb{Z}_{\gcd(N_c, N_f)}} \times (\mathbb{Z}_{2N_f})_A.$$

A: $SU(N_f)_V$; B: $\mathbb{Z}_{\gcd(N_c, N_f)}$

$$\mathcal{Z}[(A,B)] \xrightarrow{(\mathbb{Z}_{2N_f})_A} \mathcal{Z}[(A,B)]\, e^{\frac{i \gcd(N_c, N_f)}{4\pi} \int B \cup B}$$

i.e. Without the infinitesimal part of $SU(N_f)_A$, we have an 't Hooft anomaly.

Applications $\Rightarrow$

- Stern phase is ruled out at $T=0$, $\mu \neq 0$. (in progress)
- $\mathbb{Z}_3$-QCD has a nontrivial phase diagram

2. New 't Hooft anomaly of massless QCD

Full symmetry of massless QCD:

$$G^{\text{full}} = \frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times (\mathbb{Z}_{2N_f})_A}{\mathbb{Z}_{N_c} \times (\mathbb{Z}_{N_f})_L \times (\mathbb{Z}_{N_f})_R \times \mathbb{Z}_2}.$$

We especially pay attention to the case $N_c = N_f$ (for simplicity) and consider the subgroup

$$G = \frac{SU(N)_V}{\mathbb{Z}_N} \times (\mathbb{Z}_{2N})_A.$$

($(\mathbb{Z}_{2N})_A$: anomaly-free subgroup of $U(1)_A$.)

$SU(N)_V$ is the vector-like flavor rotation

$$q_{cf} \longmapsto U_{ff'} q_{cf'}.$$

$U \in SU(N)_V$ contains the center element:

$$U = e^{\frac{2\pi i}{N}} 1_N.$$

However,

$$q_{cf} \longmapsto e^{\frac{2\pi i}{N}} q_{cf}$$

is contained in the gauge group $SU(N)$ color.

$\therefore$ Gauge invariance says that the "correct" vector-like flavor symmetry is

$$\frac{SU(N)}{\mathbb{Z}_N}.$$

To see the 't Hooft anomaly of $G = \frac{SU(N)}{\mathbb{Z}_N} \times (\mathbb{Z}_{2N})_A$, we gauge $\frac{SU(N)}{\mathbb{Z}_N}$ and see that $(\mathbb{Z}_{2N})_A$ is gone.

Note. $SU(N)$ gauge field A : 1-form.
We want to gauge $\frac{SU(N)}{\mathbb{Z}_N}$, not $SU(N)$!

$\frac{SU(N)}{\mathbb{Z}_N}$ gauge field (A, B)
- A : $SU(N)$ gauge field (1-form)
- B : $\mathbb{Z}_N$ 2-form gauge field.

Effect of B

After gauging $\frac{SU(N)}{\mathbb{Z}_N}$, the gauge group is

$$\frac{SU(N)_{\text{color}} \times SU(N)_{\text{flavor}}}{(\mathbb{Z}_N)_{\text{diagonal}}}.$$

B generates the transformation

$$\begin{cases} W_{\text{color}}(C) = \operatorname{tr}(\mathcal{P} e^{i\oint a}) \longmapsto e^{\frac{2\pi i}{N}} W_{\text{color}}(C) \\ W_{\text{flavor}}(C) = \operatorname{tr}(\mathcal{P} e^{i\oint A}) \longmapsto e^{-\frac{2\pi i}{N}} W_{\text{flavor}}(C). \end{cases}$$

As a result, the topological charge $\in \frac{1}{N}\mathbb{Z}$:

$$Q_{\text{top}} = \frac{1}{8\pi^2}\int \operatorname{tr}\left((da+a^2)\wedge(da+a^2)\right) \overset{\text{mod } 1}{\equiv} \frac{iN}{2(2\pi)^2}\int B\wedge B \in \frac{1}{N}\mathbb{Z}_N$$

$\therefore$ Under the discrete axial trans. $\psi \longmapsto e^{\frac{2\pi i}{2N}\gamma_5}\psi$, $\left(\bar\psi \mapsto \bar\psi e^{\frac{2\pi i}{2N}\gamma_5}\right)$,

we get

$$Z[(A,B)] \longmapsto Z[(A,B)] \underbrace{e^{i2\cdot\frac{2\pi}{2N}N Q_{\text{top}}}}_{e^{i2\pi Q_{\text{top}}} = e^{i\frac{N}{4\pi}\int B\wedge B}}.$$

$\therefore$ $\frac{SU(N)_{\text{flavor}}}{\mathbb{Z}_N} \times (\mathbb{Z}_{2N})_A$ has an 't Hooft anomaly.

Application 1. (in progress) : Stern phase

Chiral symmetry breaking $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$ is believed to happen by

$$\langle \bar\psi \psi \rangle \neq 0.$$

However, it is theoretically possible (a priori) that $(\mathbb{Z}_{2N_f})_A$ is unbroken:

$$\langle \bar\psi\psi \rangle = 0, \quad \sum_a \langle (\bar\psi(1-\gamma_5)T_a\psi)(\bar\psi(1+\gamma_5)T_a\psi)\rangle \neq 0.$$

(Stern phase)

The above anomaly tells that if the Stern phase is realized, there must be another degeneracy of ground states in addition to (N_f^2-1) NG bosons.

(This ... is valid for $M \neq 0$. At $M=0$ it is already ... (Komargodski-Komargodski))

Application 2 : $\mathbb{Z}_N$-QCD

Compactify 1 direction : $S^1 \times \mathbb{R}^{1,2}$.
(S^1: x_4, $\mathbb{R}^{1,2}$: x)

$$\xi_f(x, x_4 + L) = e^{\frac{2\pi i}{N} f}\, \xi_f(x, x_4).$$

Color-flavor center symmetry

4-th Derivative $\sum_f \bar{\xi}\gamma_4 D_4 \xi = \sum_f \bar{\xi}\gamma_4 (\partial_4 + a_4 + \frac{2\pi i}{NL} f)\xi_f$.

This is inv. under

$$\begin{cases} a_4 \to a_4 + \frac{2\pi i}{NL}, \\ \xi_f \to \xi_{f+1} \end{cases}$$

i.e. there is the $\mathbb{Z}_N$ symmetry related to the Polyakov loop:

$$\begin{cases} \Phi = \mathrm{tr}(P e^{i\int_{S^1} a_4}) \to e^{\frac{2\pi i}{N}} \Phi. \\ \xi_f \to \xi_{f+1}. \end{cases}$$

∴ We have the following symmetry as a QFT on $\mathbb{R}^{1,2}$:

$$\mathbb{Z}_N \times \frac{U(1)^{N-1}}{\mathbb{Z}_N} \times (\mathbb{Z}_{2N})_A.$$

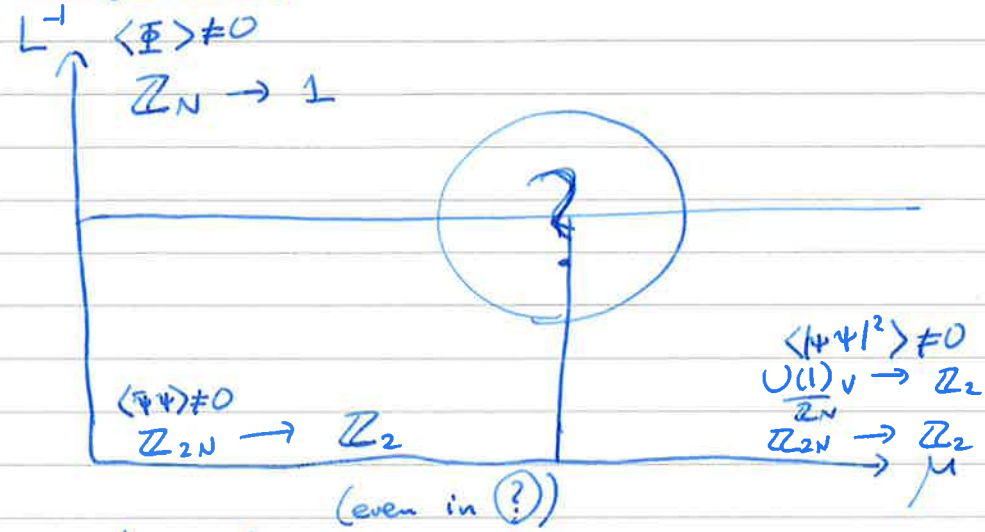
$B^{(1)}$ ↗ ; ↑ $(A, B^{(2)})$.

$$Z_{\mathbb{Z}_N\text{-QCD}}[B^{(1)}; (A, B^{(2)})] = Z_{QCD}[(A, \underbrace{B^{(1)} \wedge L^{-1} dx^4}_{B''} + B^{(2)}]$$

$$\therefore \mathcal{A} = \frac{iN}{4\pi} \int B \wedge B = \frac{iN}{2\pi} \int B^{(1)} \wedge B^{(2)}.$$

So, new't Hooft anomaly of massless QCD on $\mathbb{R}^{1,3}$ implies the 't Hooft anomaly of $\mathbb{Z}_N$-QCD on $\mathbb{R}^{1,2}$.

Phase diagram of $\mathbb{Z}_N$ - QCD



At any L and μ, some of the symmetry must be broken to match the anomaly.

Conclusion

We find a new 't Hooft anomaly for massless 3-flavor QCD involving $SU(N)_V / \mathbb{Z}_N \times (\mathbb{Z}_{2N})_A$.

$\Rightarrow$ New nonperturbative constraints on possible QCD dynamics

Applications:
- Stern phase
- $\mathbb{Z}_3$-QCD.